PART 1: QUESTIONS

Name:________________________________ **Age:**_____ **Id:**_______________ **Course:**__________

Algebra II - Exam 3 **Lesson: 7-10**

Instructions:

- Please begin by printing your Name, your Age, your Student Id , and your Course Name in the box above and in the box on the solution sheet.
- You have 90 minutes (class period) for this exam.
- You can not use any calculator, computer, cellphone, or other assistance device on this exam. However, you can set our flag to ask permission to consult your own one two-sided-sheet notes at any point during the exam (You can write concepts, formulas, properties, and procedures, but questions and their solutions from books or previous exams are not allowed in your notes).
- Each multiple-choice question is worth 5 points and each extra essay-question is worth from 0 to 5 points. (Even a simple related formula can worth some points).
- Set up your flag if you have a question.
- Relax and use strategies to improve your performance.

Exam Strategies to get the best performance:

- Spend 5 minutes reading your exam. Use this time to classify each Question in (E) Easy, (M) Medium, and (D) Difficult.
- Be confident by solving the easy questions first then the medium questions.
- Be sure to check each solution. In average, you only need 30 seconds to test it. (Use good sense).
- Don't waste too much time on a question even if you know how to solve it. Instead, skip the question and put a circle around the problem number to work on it later. In average, the easy and medium questions take up half of the exam time.
- Solving the all of the easy and medium question will already guarantee a minimum grade. Now, you are much more confident and motivated to solve the difficult or skipped questions.
- Be patient and try not to leave the exam early. Use the remaining time to double check your solutions.

1. An exponential function with base b is defined as:

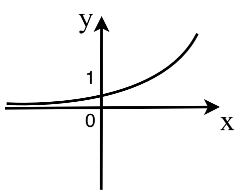
a) $f(x) = b^x$, where $b > 0$ and $b \neq 1$
b) $f(x) = b^x$, where $b \in \mathbb{R}$ and $b \neq 1$
c) $f(x) = b^x$, where $b > 0$
d) $f(x) = b^x$, where $b \in \mathbb{R}$
e) None of the above.

Solution: a

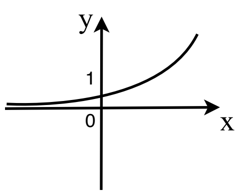
An exponential function with base b is defined as:
$f(x) = b^x$, where $b > 0$ and $b \neq 1$.

2. The exponential function $y = b^x$ has the following graph:

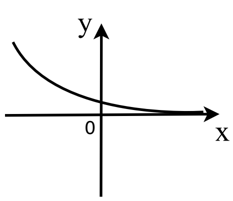
II. Case: $b < 0$



II. Case: $b > 0$



III.Case: $b < 0$

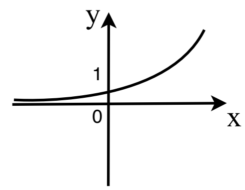


a) Only I and II are correct
b) Only I and III are correct
c) Only II and III are correct
d) I, II, and III are correct
e) None of the above.

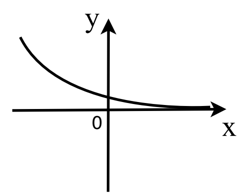
Solution: c

The exponential function $y = b^x$ has the following graph:

I. Case: $b > 0$



II. Case: $b < 0$



3. Euler number is:

a) an important constant e = 2.71 in calculus like π = 3.14 in trigonometry.
b) an important constant e = 3.14 in calculus like π = 2.71 in trigonometry.
c) an important constant e = 2.71 used in calculus and it is much more important than π = 3.14 used in trigonometry.
d) an important constant e = 2.71 used in calculus and it is much less important than π = 3.14 used in trigonometry.
e) None of the above.

Solution: a

As m increases, the expression $\left(1 + \dfrac{1}{m}\right)^m$ approaches the value to e =2.71 (Euler number).

For example, the exponential function $y = e^x$ is widely used in physics, chemistry, engineering, biology, economics, and mathematics. In calculus, $y = e^x$ is the only function whose derivative and integral is itself.

Function	Derivative	Integral
$y = e^x$	$\dfrac{dy}{dx} = e^x$	$\int e^x dx = e^x + c$

Both e and π are very important constants in math, and their usability depend on the context.

4. Let $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where

A: Compound amount

P: Principal amount

r: Interest rate per annum (use decimal in the formula)

n: Number of interest periods per year

t: Number of years.

If $400 is invested at 40% per year with interest compounded annually. What is the compound amount after 4 years?

a) $400(1+\frac{0.4}{12})^4$

b) $400(1+\frac{0.4}{12})^{12*4}$

c) $400(1+0.4)^{12*4}$

d) $400(1+0.4)^4$

e) None of the above.

Solution: d

P: Principal amount = 400 dollars

r: Interest rate per annum (use decimal in the formula) = 0.4

n: Number of interest periods per year = 1 period

t: Number of years = 4 years.

Then:

A: Compound amount = $400(1+\frac{0.4}{1})^{1*4} = 400(1+0.4)^4$.

5. A basic exponential growth and decay function is written by the following formula:

a) $N(t) = N_o e^t$
b) $N(t) = e^{kt}$
c) $N(t) = e^t$
d) $N(t) = N_o e^k$
e) None of the above.

Solution: e

A basic exponential growth and decay function is written by:

$N(t) = N_o e^{kt}$, where:

N(t): Quantity in a time t

No : Initial quantity ($t = 0$)

Growth Constant ($k > 0$)

Decay Constant ($k < 0$)

t: Number of periods.

6. A tank has 200 fish initially. The constant of growth of the fish population is $k = \frac{1}{2}$ per day. How many fishes will exist after 2 days? Hint: $N(t) = N_0 e^{kt}$, where e = 2.71.

a) 135 fishes
b) 271 fishes
c) 542 fishes
d) 813 fishes
e) None of the above.

Solution: c

$N(t) = N_0 e^{kt}$
$N(2) = 200e^{\frac{1}{2}*2}$
$N(2) = 200 * e$
$N(2) = 542$ fishes

7. A bacteria culture starts with 100 bacteria, and in 2 hours has grown to 300. The formula for the amount of bacteria after t hours is:

Hint: $N(t) = N_0 e^{kt}$

a) $N(t) = 100(2)^{\frac{t}{2}}$
b) $N(t) = (100)2^{\frac{t}{3}}$
c) $N(t) = (100)3^{\frac{t}{2}}$
d) $N(t) = (100)3^{\frac{t}{3}}$
e) None of the above.

Solution: c

For $t = 0$ => 100 bacteria
$N(t) = N_0 e^{kt}$
$N(0) = N_0 e^{k0}$
$100 = N_0 e^{0}$
$N_0 = 100$ bacteria

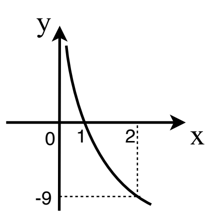
For $t = 2$ h => 300 bacteria
$N(2) = 100e^{k2}$
$300 = 100e^{k2}$
$3 = e^{k2}$
$ln(3) = ln(e^{k2})$
$k = \dfrac{ln3}{2}$,

then:

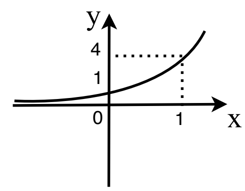
$N(t) = 100e^{\frac{\ln 3}{2}t}$
$N(t) = 100e^{ln3^{\frac{t}{2}}}$
$N(t) = (100)3^{\frac{t}{2}}$

8. The graph of $y = \left(\dfrac{1}{4}\right)^x$ is:

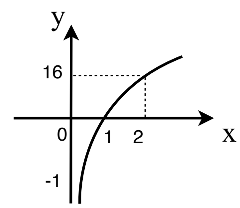
a)



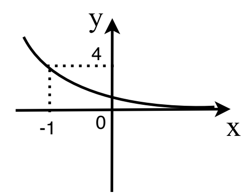
b)



c)



d)



e) None of the above.

Solution: d

9. The logarithmic function is:

a) NOT the inverse of an exponential function for $b > 0$ and $b \neq 1$
b) NOT the inverse of an exponential function for $b > 0$
c) the same of an exponential function for $b > 0$ and $b \neq 1$
d) the same of an exponential function for $b > 0$
e) None of the above.

Solution: a

Given $b > 0$ and $b \neq 1$ for $f(x) = b^x$ and $f^{-1}(x) = \log_b x$ we have:

Show that $f^{-1}[f(x)] = x$.

$f^{-1}[f(x)] = \log_b b^x = x$

Show that $f\left[f^{-1}(x)\right] = x$.

$f\left[f^{-1}(x)\right] = b^{\log_b x} = x.$

Thus, the logarithmic function is the inverse of an exponential function for $b > 0$ and $b \neq 1$.

10. An important implication between logarithmic function and exponential function is:

a) $y = \log_x b \Leftrightarrow x = b^y$ for For $b > 0$ and $b \neq 1$
b) $y = \log_b x \Leftrightarrow x = b^y$ for For $b < 0$ and $b \neq 1$
c) $y = \log_x b \Leftrightarrow x = b^y$ for For $b > 0$ and $b \neq 1$
d) $y = \log_b x \Leftrightarrow x = b^y$ for For $b > 0$ and $b \neq 1$
e) None of the above.

Solution: e

The logarithmic function is the inverse of an exponential function for $b > 0$ and $b \neq 1$.

Then, $y = \log_b x \Leftrightarrow x = b^y$

11. Given $y : A \to B$ such that $y = e^{2x}$. Then $y^{-1} : B \to A$ is:

($\mathbb{R}^+$ => Positive reals numbers)

a) $y = \log_e \sqrt{x}$ where for $A = \mathbb{R}$ and B $= \mathbb{R}^+$
b) $y = \log_e \sqrt{x}$ where for $A = \mathbb{R}$ and B $= \mathbb{R}$
c) $y = \log_e x$ where for $A = \mathbb{R}$ and B $= \mathbb{R}^+$
d) $y = \log_e x$ where for $A = \mathbb{R}$ and B $= \mathbb{R}$
e) None of the above.

Solution: a

$y = e^{2x}$ $(x \Leftrightarrow y)$
$x = e^{2y}$
$2y = \log_e x$
$y = \dfrac{\log_e x}{2}$
$y = \log_e \sqrt{x}$

Note that $A = \mathbb{R}$ and B $= \mathbb{R}^+$ for the functions be bijective.

12. The domain of the logarithm function $y = \log_{(1-x)}(x+4)$ is:

a) D $=\{x \in \mathbb{R} \,/ -4 < x < 1\}$
b) D $= \{x \in \mathbb{R} \,/ -4 \leq x \leq 0$ or $0 \leq x \leq 1\}$
c) D $=\{x \in \mathbb{R} \,/ -4 < x < 0$ or $0 < x < 1\}$
d) D $= \{x \in \mathbb{R} \,/ -4 \leq x \leq 1\}$
e) None of the above.

Solution: c

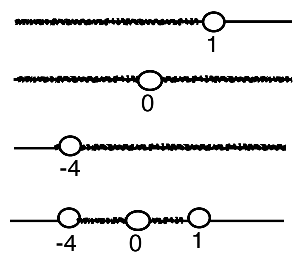
$y = \log_{(1-x)}(x+4)$

Existence: $1 - x > 0 \Rightarrow x < 1$

$1 - x \neq 1 \Rightarrow x \neq 0$

$x + 4 > 0 \Rightarrow x > -4$

D $= \{x \in \mathbb{R} \,/ -4 < x < 0$ or $0 < x < 1\}$



13. Absolute value function $f : \mathbb{R} \to \mathbb{R}$: is defined by:

a) $f(x) = \left\{ \begin{array}{r} x\text{, if } x > 0 \\ -x\text{, if } x < 0 \end{array} \right\}$

b) $f(x) = \left\{ \begin{array}{r} -x\text{, if } x > 0 \\ x\text{, if } x < 0 \end{array} \right\}$

c) $f(x) = \left\{ \begin{array}{r} -x\text{, if } x \geq 0 \\ x\text{, if } x < 0 \end{array} \right\}$

d) $f(x) = \left\{ \begin{array}{r} x\text{, if } x \leq 0 \\ -x\text{, if } x > 0 \end{array} \right\}$

e) None of the above.

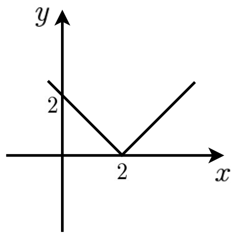
Solution: e

The absolute value function $f : \mathbb{R} \to \mathbb{R}$: is defined by:

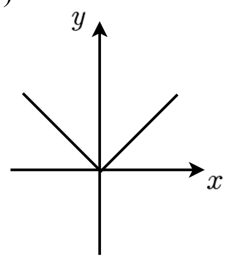
$f(x) = \left\{ \begin{array}{r} x\text{, if } x \geq 0 \\ -x\text{, if } x < 0 \end{array} \right\}$

14. The graph of $y = |x|$ is:

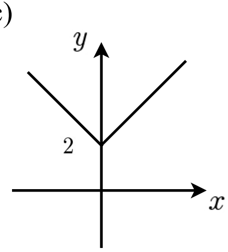
a)



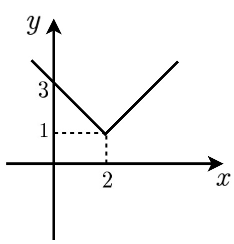
b)



c)



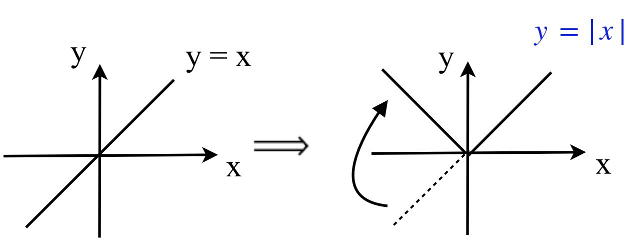
d)



e) None of the above.

Solution: b

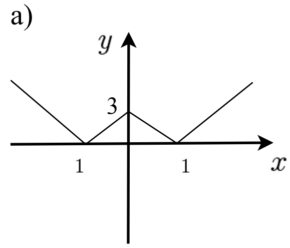
The graph of $y = |x|$ is:



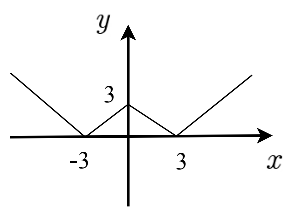
Note: The negative section becomes positive.

15. The graph of $y = ||x| - 3|$ is:

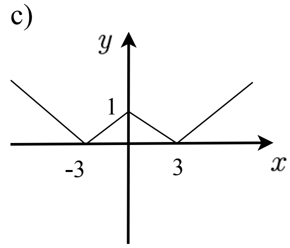
a)



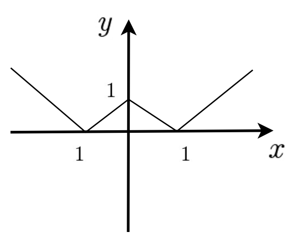
b)



c)



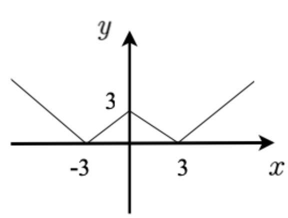
d)



e) None of the above.

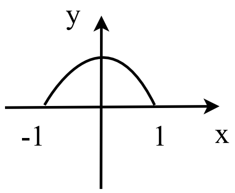
Solution: b

The graph of $y = ||x| - 3|$ is:

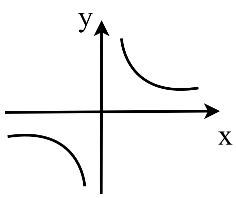


16. Given:

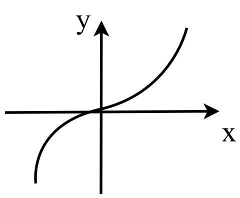
I. a. $y = \dfrac{1}{x}$



II. b. $y = x^3$



III. c. $y = \sqrt{1 - x^2}$



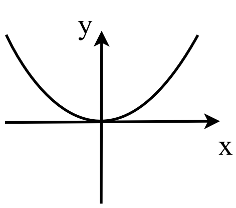
The correct association is:

a) I-a II-c III-b
b) I-c II-a III-b
c) I-b II-c III-a
d) I-c II-b III-a
e) None of the above.

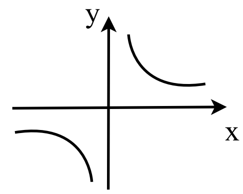
Solution: b

The correct association is:

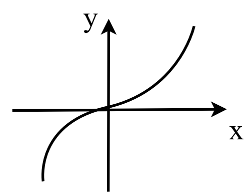
I. $y = x^2$



II. $y = \dfrac{1}{x}$

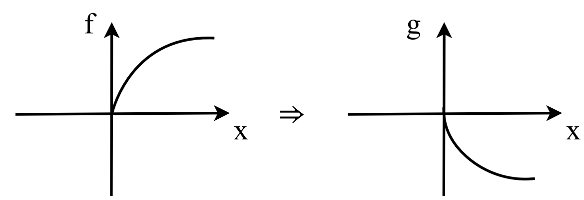


III. $y = x^3$



17. Given the graphs f and g.

$f = \sqrt{x}$ $g = ?$



Then,

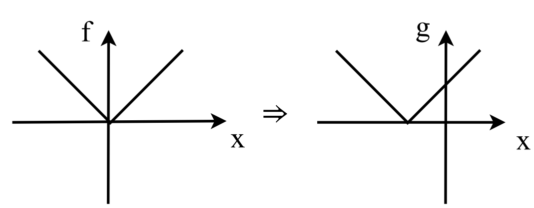
a) $g = \sqrt{x} - 1$
b) $g = -\sqrt{x}$
c) $f = \sqrt{x - 1}$
d) $f = \sqrt{-x}$
e) None of the above.

Solution: b

The graph f flip over x axis then $g = -\sqrt{x}$.

18. Given the graphs f and g.

$f = |x|$ $g = ?$



Then,

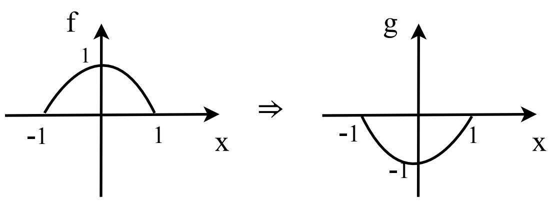
a) $g = |x| - 1$
b) $g = -|x|$
c) $g = |x - 1|$
d) $g = |-x|$
e) None of the above.

Solution: e

The graph f moves left 1 unit then $g = |x + 1|$.

19. Given the graphs f and g.

$f = \sqrt{1 - x^2}$ $g = ?$



Then,

a) $g = \sqrt{1 - x^2} - 1$
b) $g = -\sqrt{1 - x^2}$
c) $g = \sqrt{1 - (x - 1)^2}$
d) $g = \sqrt{1 - (-x)^2}$
e) None of the above.

Solution: b

The graph f flip over x axis then $g = -\sqrt{1 - x^2}$.

20. Given:

I. In Backward Technique is important to scratch graphs very fast.

II. Backward Technique uses the graph movement to graph each function from the basic function to f(x).

III. Backward Technique transforms the function f(x) to a simpler one until you have a basic function.

a) Only I and II are correct
b) Only I and III are correct
c) Only II and II are correct
d) I, II, and II are correct
e) None of the above.

Solution: d

Backward Technique is important to scratch graphs very fast. This technique transforms the function f(x) to a simpler one until you have a basic function, and it uses the graph movement to graph each function from the basic function to f(x).

Thus, I, II and III are correct.

PART 2: SOLUTIONS

Consulting

Name:______________________________ **Age:**_____ **Id:**_______________ **Course:**__________

Multiple-Choice Answers

Questions	A	B	C	D	E
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
17					
18					
19					
20					

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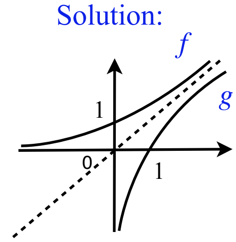
	Points	Max
Multiple Choice		100
Extra Points		25
Consulting		10
Age Points		25
Total Performance		160
Grade		A

Extra Questions

21. Show that $f = 10^x$ and $g = log_{10}x$ together in the same x and y axis (Hint: Inverse functions are symmetric to the straight line $y = x$).

Solution:



22. Graph $y = \left|x^2 - 10x + 24\right|$

Solution:

$y = x^2 - 10x + 24$

$S = \dfrac{-b}{a} \Rightarrow S = \dfrac{-(-10)}{(1)} \Rightarrow S = 10$

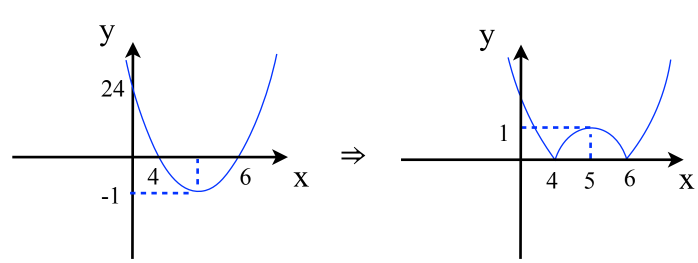
$P = \dfrac{c}{a} \Rightarrow P = \dfrac{24}{(1)} \Rightarrow P = 24$

+1 +24
+3 +8
+4 +6 (Roots)

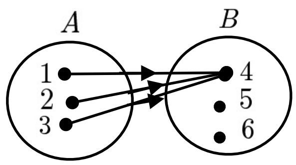
$x_v = \dfrac{-b}{2a} \Rightarrow x_v = 5$

$y_v = x_v^2 - 10x_v + 24 \Rightarrow y_v = -1$

$y = x^2 - 10x + 24$ $\quad$ $y = \left|x^2 - 10x + 24\right|$



23. Given $f : A \to B$ be a function. Find the domain D_f, codomains CD_f, and image Im_f of f.



Solution:

$D_f = \{1, 2, 3\}$ $CD_f = \{4, 5, 6\}$ $Im_f = \{4, 5\}$.

24. Given $f(x) = x + 6$. Find $f^{-1}(x)$ and proof that

$f\left[f^{-1}(x)\right] = x$.

Solution:

$f = x + 6 \ \ (x \Leftrightarrow f)$
$x = f + 6$
$f = x - 1$
$f^{-1}(x) = x - 6.$

Proof: $f\left[f^{-1}(x)\right] = x$

$f\left[f^{-1}(x)\right] = \left[f^{-1}(x)\right] + 6$
$f\left[f^{-1}(x)\right] = \left[x - 6)\right] + 6$
$f\left[f^{-1}(x)\right] = x.$

25. Given:

$H_{real} + H_{mirror} = 12$, where:

H_{real} is the current time.
H_{mirror} is the time showed by a mirror.

In a beautiful morning, John sees in his mirror the Big Ben showing 2 o'clock. What time is it in London?

Solution:

$H_{real} + H_{mirror} = 12$
$H_{real} + 2 = 12$
$H_{real} = 12 - 2$
$H_{real} = 10$. Thus, it is 10:00 AM in London.